Assignment 2

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1 Problem 1

In the lecture we derived a vertical eigenvalue problem for an ocean with a flat bottom and a constant Brunt-Väisälä frequency. In the lecture we took a look at the decomposition of the θ-function which was assumed to describe the shape of the volume force. In the following we will do an analogous decomposition, but we assume another function to describe the shape of the volume force:

$$f(z) = e^{\frac{z}{H}}, \tag{1}$$

where H, $H > 0$, is the water depth.

(a) Compute the coefficients a_0 and a_n which are given by:

$$a_n = \frac{1}{H}\int_{-H}^{0} f(z)F_n(z)dz. \tag{2}$$

Use the eigenfunctions derived in the lecture:

$$F_0 = 1 \tag{3}$$

$$F_n(z) = \sqrt{2}(-1)^n \cos\left(n\pi\frac{z}{H}\right) \tag{4}$$

(b) $f(z)$ can be rebuilt by summation of the product of the coefficients and eigenfunctions:

$$f(z) = \sum_{n=0}^{N} a_n F_n(z). \tag{5}$$

Write a function which can construct $f(z)$ from a_n and F_n. Your code does not need to calculate the eigenvalues, eigenfunctions or coefficients itself! Use your results from (a). We suggest do split the whole problem into 3 functions:

```
a_n(N,H),
F_n(N,H,z),
do_sum(a_n,F_n,z_array,N,H),
```

where `a_n` is a function which returns an array of length N with the coefficients a_n, `F_n` a function which returns an array of length N of the eigenfunctions at depth z and `do_sum` a function which does the summation given in eq. 5 and returns an array of length of `z_array`. Useful python knowledge:

- You can pass functions to another function, i.e. `a_n` and `F_n` in `do_sum(a_n,F_n,z_array,N,H` could be a function which is then used.
- `np.sum(X)` automatically does a summation over a given n-d array X

These are just hints and don't have to be used.

(c) Plot $f(z)$ and compare it to the results of your `do_sum` function for $N = 1, 4, 8, 12, 16, 20$ and $H = 1000$m. What do you see?